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Wilson loops, confinement, and phase transitions in large N gauge theories from supergravity

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Abstract: We use the recently proposed supergravity approach to large N gauge theories to calculate ordinary and spatial Wilson loops of gauge theories in various dimensions. In this framework we observe an area law for spatial Wilson loops in four and five dimensional supersymmetric Yang-Mills at finite temperature. This can be interpreted as the area law of ordinary Wilson loops in three and four dimensional non-supersymmetric gauge theories at zero temperature which indicates confinement in these theories. Furthermore, we show that super Yang Mills theories with 16 supersymmetries at finite temperature do not admit phase transitions between the weakly coupled super Yang Mills and supergravity regimes. This result is derived by analyzing the entropy and specific heat of those systems as well as by computing ordinary Wilson loops at finite temperature. The calculation of the entropy was carried out in all different regimes and indicates that there is no first order phase transition in these systems. For the same theories at zero temperature we also compute the dependence of the quark anti-quark potential on the separating distance.

Keywords: Confinement, Black Holes in String Theory, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

In the last couple of months it has become clear that supergravity is a useful tool to study the large N limit of field theories [1]. Related work and many new results can be found in refs. [2]–[35]. In the present paper we follow this approach and consider phase transitions, Wilson loops and confinement in supersymmetric and non-supersymmetric field theories using supergravity. We use the corresponding extremal and near extremal supergravity solutions in the decoupling limit [1, 10] and study Wilson loops. To consider the supersymmetric theories at zero (finite) temperature we follow [37, 38] ([39, 40]) and use the Nambu-Goto action in the background of an extremal (near-extremal) D-brane solution to calculate the space-time Wilson loop. We find that for the field theories with maximal supersymmetries there is no finite temperature phase transition between the SYM and SUGRA regimes [10]. This we also check via a direct entropy consideration which shows that the entropy matches (up to numerical coefficients which we do not calculate) across the different domains. Since the entropy seems to match in all the different regimes of the theory it is most likely that there is no first order phase transition. This does not exclude higher order phase transitions which are known to occur in various cases as we enter the eleven dimensional M theory regime. In those cases a "localization" phase transition takes place as discussed in [10]. To consider non-supersymmetric theories at zero temperature we follow [46] and consider the spatial Wilson loop in the background of Euclidean near-extremal Dp-brane solutions. When the spatial size is much larger then $1/T$ (where T is the Hawking temperature of the near-extremal solution) the effective low energy theory reduces effectively to a p dimensional non-supersymmetric theory. Therefore, the spatial Wilson loop gives us the energy between a quark and an anti-quark of the p dimensional theory at zero temperature. Using this approach we find an area law behavior for non-supersymmetric YM in three and four dimensions in the large N limit.

The paper is organized as follows: in section 2 we briefly describe following [10] the supergravity solutions of N coincident Dp-branes in the field theory limit that was introduced in [1]. In section 3 we use the supergravity approach to confirm the confinement behavior of non-supersymmetric YM theory in three (four) dimensions. We consider the Euclidean theory of N D3- (D4-) branes with $R^3 \times S^1$ ($R^4 \times S^1$) world volume. In the limit of small radius of the S^1 circle, imposing appropriate boundary conditions, we end up with a three (four) dimensional pure YM theory. In this setup we compute the spatial Wilson loop and show that it admits an area law behavior. Section 4 is devoted to analyzing the entropy and the specific heat of the various theories with 16 supersymmetries. It is shown that there is no phase transition in the SYM, supergravity and other domains of the Dp-brane systems. In section 5 we present the derivation of ordinary Wilson lines (along one space and one time directions) for these theories at zero temperature. This includes the derivation in the ten dimensional supergravity regime, as well as in the extensions to other energy domains. In section 6 the finite temperature behavior of these systems is deduced from the supergravity description. It is shown that for $p = 1 \dots 4$ they admit a similar behavior as the one discovered in [39], [40].

2. A very brief review of the theories with 16 supersymmetries

In [10] systems of N coincident extremal Dp-branes were analyzed in the decoupling limit [1]

$$g_{YM}^2 = (2\pi)^{p-2} g_s \alpha'^{(p-3)/2} = \text{fixed}\,, \qquad \alpha' \to 0\,, \qquad U \equiv \frac{r}{\alpha'} = \text{fixed}\,, \tag{2.1}$$

where $g_s = e^{\phi_\infty}$, and g_{YM} is the coupling constant of the $p+1$ dimensional $U(N)$ SYM theory (with sixteen supercharges) that lives on the N Dp-branes. In the SYM picture U corresponds to finite Higgs expectation value associated with a $U(N+1) \rightarrow U(N) \times U(1)$ symmetry breaking. The effective coupling of the SYM theory is $g_{eff}^2 = g_{YM}^2 N U^{p-3}$. Perturbation theory can be trusted in the region

$$g_{eff} \ll 1 \,. \tag{2.2}$$

The type II supergravity solution describing N coincident extremal Dp-branes can be trusted if the curvature in string units and the effective string coupling are small. These conditions yield

$$1 \ll g_{eff}^2 \ll N^{\frac{4}{7-p}} \,. \tag{2.3}$$

which translate for $p < 3$ to the following range of the energy scale U

$$(g_{YM}^2 N)^{1/(3-p)} N^{4/(p-3)(7-p)} \ll U \ll (g_{YM}^2 N)^{1/(3-p)} \,. \tag{2.4}$$

For $p > 3$ the $\ll$ signs are replaced with $\gg$ ones. In the supergravity description U is the radial coordinate.

In general, there are other regions (which will play a role below) which take over when the dilaton becomes large. We do not describe these regions here since they depend on p and cannot be describes in this concise manner. For details see [10].

3. Area law

In this section we use the supergravity approach to field theory and find confinement in non-supersymmetric theories in three and four dimensions.

3.1. Area law for three dimensional YM theory

We consider the Wilson loop along two space directions in the case of the near extremal D3 brane solution. We will find that it shows area law behavior as in the R^3 case in ref. [46]. We shall take one direction, Y, to be large and the other direction, L, to be finite. In the limit $Y \rightarrow \infty$ we consider configurations which are invariant under translation in the Y direction.

To describe the theory at finite temperature we go to a Euclidean description and compactify the time direction on a circle with period $\beta = T^{-1}$. For large temperature this circle is small and the theory becomes effectively the Euclidean description of a three dimensional field theory with gauge coupling given by

$$g_{YM_3}^2 = g_{YM_4}^2 T \,. \tag{3.1}$$

Since we choose boundary conditions on the circle such that supersymmetry is broken, the fermions and scalar fields are heavy with masses of the order T and $g_{YM4}^2 T$,

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respectively [46]. Therefore, at large distances, $L \gg \beta$, we obtain zero temperature non-supersymmetric Yang-Mills theory in three dimensions. Confinement and area law for the Wilson loop are expected. We will derive this momentarily by calculating a spatial Wilson loop of the "compactified" four dimensional theory. Since one of the space directions becomes the Euclidean time direction of the three dimensional theory this Wilson line is an order parameter of the theory. Note that the area law behavior, found in this section, does not imply confinement in the 3+1 dimensional theory with temperature.

Confinement is expected to appear in the limit where $LT \gg 1$ i.e. the distance between the quarks is much larger than the size of the circle. On the other hand if we consider small distances we are back in the zero temperature 3+1 dimensional theory studied in [37]. Thus, our description interpolates between confinement in 2+1 dimensions ($LT \gg 1$) and Coulomb behavior in $\mathcal{N} = 4$ YM in 3+1 dimensions ($LT \ll 1$).

The metric of near extremal D3 branes in the large N limit is

$$\begin{aligned} ds^2 &= \alpha' \left\{ \frac{U^2}{R^2}[-f(U)dt^2 + dx_i^2] + R^2 f(U)^{-1}\frac{dU^2}{U^2} + R^2 d\Omega_5^2 \right\} \\ f(U) &= 1 - U_T^4/U^4 \\ R^2 &= \sqrt{4\pi g N}, \qquad\qquad U_T^4 = \frac{2^7}{3}\pi^4 g^2 \mu\,, \end{aligned} \tag{3.2}$$

where μ is the energy density. Thus, the relevant action for the spatial Wilson loop is

$$S = \frac{Y}{2\pi}\int dx \sqrt{\frac{U^4}{R^4} + \frac{(\partial_x U)^2}{1 - U_T^4/U^4}}\,. \tag{3.3}$$

The distance between the quark and the anti-quark is

$$L = 2\frac{R^2}{U_0}\int_1^\infty \frac{dy}{\sqrt{(y^4-1)(y^4-\lambda)}}\,, \tag{3.4}$$

where $\lambda = U_T^4/U_0^4 < 1$ and U_0 is the minimal value of U. Notice that in the limit $U_0 \to U_T$ ($\lambda \to 1$) we get $L \to \infty$. The energy is

$$E = \frac{U_0}{\pi}\int_1^\infty dy \left(\frac{y^4}{\sqrt{(y^4-1)(y^4-\lambda)}} - 1 \right) + \frac{U_T - U_0}{\pi}\,, \tag{3.5}$$

where, as was explained in [39], the subtraction is at the horizon. We are after the large L limit behavior. Thus we need to take the limit $\lambda \to 1$. In this limit the main contribution to the integrals in (3.4) and (3.5) comes from the region near $y = 1$. Therefore, we get for large L

$$E = T_{QCD} L. \tag{3.6}$$

The tension of the QCD string is

$$T_{QCD} = \frac{\pi}{2} R^2 T^2 \,, \tag{3.7}$$

where we have used the relation [39] $U_T = \pi R^2 T$.

It is important to emphasize that the theory for which we obtain the area law is not YM in three dimensions but $\mathcal{N} = 4$ at four dimensions compactified on a circle. The string tension which we derive (3.7) "knows" about the four dimensional origin of the theory. The reason is as follows. From (3.7) we see that the mass of the excitations of the QCD strings is $M_s = RT$. The mass associated with the compactification is $M_c = T$. To trust the supergravity solution we need $R \gg 1$ [1]. Thus $M_s \gg M_c$ and so the QCD strings probe distances which are much smaller then the radius of compactification.

Some speculations about string theory at large curvature can be made. For YM in three dimensions we expect to have $T_{QCD} \sim g_3^4$ which in our notation is $\sim R^8 T^2$ and not $\sim R^2 T^2$ as in (3.7). The system which we are considering here is a good approximation to YM in three dimensions if $R \ll 1$ (because then the masses of the excitations of the QCD string is lighter then the masses associated with the compactification). The supergravity solution cannot be trusted in that region. However, since the solution is near-extremal it makes some sense to speculate that the exact string theory solution has the same form but with corrected harmonic functions. In that case in the limit which we are studying we will get for the exact string theory solution an AdS space times a sphere where the radius of the AdS is not R^2 but a function of R^2. The fact that for YM in 3d we expect to get R^8 for $R^2 \ll 1$ implies that the exact string theory solution interpolates between R^2 at large R^2 and R^8 at small R^2.

3.2. Area law in four dimensional YM theory

The approach of the previous section to confinement can be generalized to obtain confinement in four dimensions from supergravity. We need to consider the supergravity solution of near-extremal D4-brane in the decoupling limit. A D4-brane is described in M-theory as a wrapped M5-brane so from the point of view of M-theory we relate the near-extremal solution of M5-brane to confinement in four dimensions as was suggested in sect. 4 of [46].

The near-extremal solution of D4-branes in the decoupling limit is [10]

$$\begin{aligned} ds^2 &= \alpha' \Bigg[\frac{U^{3/2}}{R_4^{3/2}} \left((1 - U_T^3/U^3) dx_0^2 + dx_1^2 + ... + dx_4^2 \right) \\ &\qquad + \frac{R_4^{3/2}}{U^{3/2}(1 - U_T^3/U^3)} dU^2 + R_4^{3/2} \sqrt{U} d\Omega_4^2 \Bigg] \,, \\ e^{\phi} &= \frac{1}{(2\pi)^2} g_5^2 \left(\frac{U^3}{R^3} \right)^{1/4} \,, \end{aligned} \tag{3.8}$$

where $R_4^{3/2} = g_5\sqrt{\frac{N}{4\pi}}$ and g_5 is the 5D SYM coupling constant.

We would like to study the spatial Wilson loop in the region where $LT \gg 1$. In this region the effective description is via a non-supersymmetric YM theory in four dimensions with coupling constant

$$g_{YM}^2 = g_5^2 T\,. \tag{3.9}$$

Unlike the supergravity solution which was used in the previous section the supergravity solution (3.8), which we use here, cannot be trusted for arbitrary U. L, is related to U by $L \sim 1/U$ and hence it is also bounded.

Before we perform the calculation of the spatial Wilson line let us first find the upper bound on LT and the bounds on g_{YM} and $g_{eff}^2 = g_{YM}^2 N$. The restrictions on U and hence on U_T, are such that the curvature in string units and the effective string coupling are small. The result of these restrictions is (2.4) [10]

$$\frac{1}{Ng_5^2} \ll U_T \ll \frac{N^{1/3}}{g_5^2}\,. \tag{3.10}$$

Therefore, the supergravity solution can be trusted only for distances

$$Ng_5^2 \gg L \gg \frac{g_5^2}{N^{1/3}}. \tag{3.11}$$

To find the bound on T we use the relation between T and U_T (the temperature can be obtained from (3.8) and $T = \frac{1}{4\pi}\frac{dg_{tt}}{dU}\Big|_{U=U_T}$)

$$T = \frac{3\sqrt{\pi}}{g_5\sqrt{N}}\sqrt{U_T}\,, \tag{3.12}$$

to get

$$\frac{1}{Ng_5^2} \ll T \ll \frac{1}{N^{1/3}g_5^2}\,. \tag{3.13}$$

From (3.9) we find that the four dimensional coupling constant is bounded by

$$\frac{1}{N} \ll g_{YM}^2 \ll \frac{1}{N^{1/3}}\,. \tag{3.14}$$

We see, therefore, that in the large N limit g_{YM} must go to zero. For the four dimensional effective coupling, g_{eff}, we have

$$1 \ll g_{eff}^2 \ll N^{2/3}\,. \tag{3.15}$$

Thus the effective four dimensional coupling constant g_{eff} must be large otherwise we cannot trust the supergravity description. Finally we turn to the bound for TL. To be able to use the supergravity results described below we need to find a region where $TL \gg 1$. From (3.11) and (3.13) we get

$$N^{2/3} \gg TL \gg \frac{1}{N^{4/3}}\,. \tag{3.16}$$

Therefore, in the large N limit there is a region for which we can trust our results. Note that unlike the 3d case, considered in the previous section, in the 4d case, for any finite N, L is bounded.

Let us now derive the area law behavior. The action for the string in this case is

$$S = \frac{Y}{2\pi} \int dx \sqrt{\frac{U^3}{R_4^3} + \frac{\partial_x U^2}{1 - U_T^3/U^3}}\,, \tag{3.17}$$

Using the same manipulations as in [37] we get

$$\begin{aligned} L &= 2\frac{R_4^{3/2}}{U_0^{1/2}} \int_1^\infty \frac{dy}{\sqrt{(y^3-1)(y^3-\lambda_4)}}\,, \\ E &= \frac{U_0}{\pi} \int_1^\infty dy \left(\frac{y^3}{\sqrt{(y^3-1)(y^3-\lambda_4)}} - 1 \right) + \frac{U_T - U_0}{\pi}\,, \end{aligned} \tag{3.18}$$

where $\lambda_4 = \frac{U_T^3}{U_0^3}$. For $TL \gg 1$ we have $(U_T - U_0)/U_0 \ll 1$ and the integrals are dominated by the region close to $y = 1$. Therefore, as in the previous section, we get

$$E = T_{YM} L\,, \tag{3.19}$$

where the string tension is

$$T_{YM} = \frac{8}{27\pi^2} g_{YM}^2 N T^2\,. \tag{3.20}$$

This agrees with known large N results if T is identified with Λ_{QCD} up to a N independent constant factor. Again, as in the previous section, in the region where we can trust the supergravity solution, the QCD string can probe distances which are much smaller than the compactification radius and hence it "knows" about the higher dimensional origin of the underlying theory (the six dimensional (0,2) theory).

3.3. 't Hooft line

In this section we calculate the energy between a monopole and an anti-monopole in the non-supersymmetric four dimensional theory obtained from the six dimensional (0,2) upon compactification. A related discussion for supersymmetric theories can be found in [41]. The supergravity background is the same as in the previous section. Namely, it is given by (3.8). As was explained in the previous section at large distances the effective theory is four dimensional along x_1, x_2, x_3, x_4. This theory being related to YM should contain monopoles. The string theory realization of the monopole is of a D2-brane ending on the D4-brane. The D2-brane is wrapped along x_0 so from the point of view of the four dimensional theory it is a point like object.

The action of a D2-branes is

$$S = \frac{1}{(2\pi\alpha')^{3/2}} \int d\tau d\sigma_1 d\sigma_2 e^{-\phi} \sqrt{\det h}\,, \tag{3.21}$$

where h is the induced metric.

For our D2-brane which is infinite along one direction and winds the x_0 direction we get

$$S = \frac{Y}{g_{YM}^2} \int dx \sqrt{\partial_x U^2 + \frac{U^3 - U_T^3}{R_4^3}}\,, \tag{3.22}$$

where we have used (3.9). Note the $1/g_{YM}^2$ factor which is expected for a monopole.

The distance between the monopole and the anti-monopole is

$$L = 2\frac{R_4^{3/2}}{U_0^{1/2}}\sqrt{\epsilon}\int_1^\infty \frac{dy}{\sqrt{(y^3-1)(y^3-1+\epsilon)}}\,, \tag{3.23}$$

where $\epsilon = 1 - (U_T/U_0)^3$. The energy (after subtracting the energy corresponding to a free monopole and anti-monopole) is

$$E = \frac{2U_0}{(2\pi)^{3/2} g_{YM}^2}\left[\int_1^\infty dy \left(\frac{\sqrt{y^3-1+\epsilon}}{\sqrt{y^3-1}}\right) - 1\right] + \frac{2(U_T - U_0)}{(2\pi)^{3/2}}\,. \tag{3.24}$$

We would like to study the system in the region where $LT \gg 1$. At that region the energy turns positive which means that it is energetically favorable for the system to be in a configuration of two parallel D2-branes ending on the horizon and wrapping x_0 which corresponds to zero energy (after the subtraction). So in the "YM region" we find no force between the monopole and the anti-monopole. Which means that there is a screening of the magnetic charge.

4. Entropy and (no)-phase transitions in theories with maximal supersymmetry

In the conformal cases a phase transition cannot take place at finite temperature. The reason is that in conformal theories there is no scale and, therefore, a finite T_c cannot appear in the theory. Put differently, when compactifying the Euclidean time direction the radius of compactification cannot be considered as large or small simply because there is no scale in the theory to compare it with. In [46] SYM on $S^3 \times S^1$ (rather than $R^3 \times S^1$ for a superconformal theory in 3+1 dimensions) was studied. The radius of the S^3 serves as a scale in the theory. Thus a phase transition is possible for this theory. Indeed, it was shown in [46] that a phase transition occurs at finite temperature. The phase transition was manifested through a change in the sign of the specific heat of the AdS black hole [43].

In this section we would like to consider the non-conformal theories with maximal supersymmetries which were studied from the supergravity point of view in [10]. These theories, being non-conformal, contain a scale which depends on g_{YM} and N. Therefore, a phase transition at finite temperature in these theories might occur. At first sight it seems that phase transitions should appear in a natural way for the non-conformal theories. The reason is that different descriptions of the theories are valid in different energy regions [10]. However, analyzing the entropy of the systems describing the theory in these different regions, we do not detect any discontinuity in the entropy and the specific heat. This is an evidence that no first order phase transition occurs in these theories. Two comments are in order: (i) We cannot exclude possible jumps in the numerical prefactor of the entropy which, if it exists, will show a phase transition. (ii) We cannot exclude higher order phase transitions between regions other than the SYM and supergravity regions. Such transitions are known to occur in various cases as we enter into the eleven dimensional M-theory regime and are associated with "localization". An example of this phenomenon is the 2+1 dimensional theory [10].

As was reviewed in section 1 there are energy regions which are common to all p. These are the SYM region and the ten dimensional supergravity region. There are other regions whose description depends on p [10]. We shall first show that first order phase transitions do not occur in the common regions. Then we shall consider the other regions.

4.1. SYM $\leftrightarrow$ 10d supergravity

In this section we consider the SYM region, the 10d supergravity region and the transition between the two regions. Similar results using a slightly different language have been found in [48].

Perturbation theory in SYM is valid as long as $g_{eff} \ll 1$. In this regime the interactions between the gluons can be neglected and the free gas approximation can be used. For a free gas in $d+1$ dimensions we have

$$E \sim nVT^{d+1}\,, \qquad S \sim nVT^{d}\,, \tag{4.1}$$

where n is the number of massless fields. So in our case $n \sim N^2$ and hence we get

$$S_{YM} \sim N^{2/(p+1)} V^{1/(p+1)} E^{p/(p+1)}\,. \tag{4.2}$$

The supergravity solution can be trusted as long as $1/g_{eff} \ll 1$. (There is also a lower bound which we shall introduce in the next subsections). The thermodynamics of the system in this region is defined by the area of the horizon of the supergravity solution which is [10],

$$S_{sg} \sim g_{YM}^{\frac{p-3}{7-p}} \sqrt{N} E^{\frac{9-p}{2(7-p)}} V^{\frac{5-p}{2(7-p)}}\,. \tag{4.3}$$

First we would like to see whether the two entropies (4.2), (4.3) are of the same order at the transition region, $g_{eff} \sim 1$. Since $g_{eff}^2 = g_{YM}^2 N U^{p-3}$ the transition is at $U \sim (g_{YM}^2 N)^{1/(3-p)}$. The relation between U and E, V is [10] $U^{7-p} \sim g_{YM}^4 E/V$. Therefore, the transition should take place at

$$E = V N^{\frac{7-p}{3-p}} g_{YM}^{-2\frac{p+1}{p-3}}\,. \tag{4.4}$$

Indeed one can check that precisely at that point one gets

$$S_{YM} \sim S_{sg} \sim V g_{YM}^{-2\frac{p}{p-3}} N^{\frac{p-6}{p-3}} . \tag{4.5}$$

Let us consider now the specific heat $c = \frac{\partial E}{\partial T}\Big|_V$. From (4.2) and (4.3) we get

$$c_{YM} \sim S_{YM} , \qquad c_{sg} \sim S_{sg} , \tag{4.6}$$

and hence at the transition $c_{YM} \sim c_{sg}$. Moreover, it is clear from (4.6) that the specific heat is positive for any temperature and hence there is no first order phase transition in the SYM and/or 10d supergravity regions for any p.

4.2. D1-brane

Equation (4.3) with $p = 1$ cannot be trusted all the way to the IR limit ($U \to 0$). For $U < g_{YM}$ the proper description is via orbifold conformal field theory [47]. Before we discuss that region we should note that for $U \sim g_{YM} N^{1/6}$ the correct description is by the S-dual system. Since the entropy is defined in the Einstein frame and the Einstein metric is invariant under S-duality the S-dual description yields the same thermodynamics.

In the region $U < g_{YM}$ the entropy should be calculated in terms of an orbifold conformal field theory. Like in the SYM case we consider the entropy in the free theory limit. The expression we find is a good approximation up to $T \sim g_{YM}/\sqrt{N}$ [10] which corresponds to $U \sim g_{YM}$.

The maximal entropy is obtained when the configuration is that of one long string whose length is $\tilde{L} = NL$, where L is the size of the system [45]. Therefore, we have

$$E \sim \tilde{L} T^2 , \qquad S \sim \tilde{L} T , \tag{4.7}$$

which gives

$$S_{orb} \sim \sqrt{NLE} . \tag{4.8}$$

Since the transition occurs at $U \sim g_{YM}$ and since $U^6 \sim g_{YM}^4 E/L$ we find that at the transition region they are of the same order,

$$S_{orb} \sim S_{sg} \sim g_{YM} \sqrt{N} L . \tag{4.9}$$

Note that $c_{orb} \sim S_{orb}$ and there is no first order phase transition at any finite temperature for SYM in $1 + 1$ dimensions.

4.3. D2-brane

Again, like in the previous section, eq.(4.3) with $p = 2$ cannot be trusted all the way to the IR limit. At the point where [10] $U \sim g_{YM}^2$ the correct description becomes the conformal theory of coinciding M2 branes with SO(8) R-symmetry. Therefore, in the region $U < g_{YM}^2$ the entropy is due to the area of a collection of N near-extremal M2-branes [44]

$$S_{M2} = \sqrt{N} V^{1/3} E^{2/3} \,. \tag{4.10}$$

One can easily check [10] that near the transition region $U \sim g_{YM}^2$

$$S_{M2} \sim S_{sg} \sim \sqrt{N} V g_{YM}^4 \,. \tag{4.11}$$

Note that $c_{M2} \sim S_{M2}$ and hence there is no first order phase transition that can be detected by the entropy . However, it was pointed out in [10] that there is a phase transition between a translation (along x_{11}) invariant solution and a localized one. This type of phase transition, very likely to be a higher order one, does not show itself in the entropy or specific heat which are continuous.

4.4. D4-branes

Equation (4.3) with $p = 4$ cannot be trusted all the way to the UV limit, $U \to \infty$. The correct description for $U > N^{1/3}/g_{YM}^2$ is via M5-branes wrapped along x_{10} [10]. The entropy is [44]

$$S_{M5} \sim \sqrt{N} V_5^{1/6} E^{5/6} \,, \tag{4.12}$$

where $V_5 = V_4 2\pi R_{10}$ is the volume of the M5-brane. Since $R_{10} = g_{YM}^2/(2\pi)^2$ (4.12) is the same as (4.3) for $p = 4$. We conclude that there is no phase transition for the (0,2) theory on a circle. Note that for the six dimensional (0,2) theory a phase transition at finite temperature cannot occur simply because the theory is conformal and hence there is no scale. What we show above is that there is no first order phase transition even if one introduces a scale via compactification.

4.5. D5-branes

In the UV there is a transition from the D5-brane solution to the NS-fivebrane solution [10]. The transition is via S-duality. Since S-duality does not change the Einstein metric and the entropy is one quarter of the area in the Einstein frame the entropy is intact.

4.6. D6-branes

The correct description of D6-branes in the UV region, $U > \frac{N}{g_{YM}^{2/3}}$, is via Schwarzschild black hole sitting on the A_{N-1} singularity [10]. To calculate the entropy one should calculate the area of the horizon taking into account the Z_N identifications (which add a factor of $1/N$ to the usual Schwarzschild result)

$$S \sim \frac{\sqrt{N} E^{3/2} g_{YM}^4}{\sqrt{V}} \,. \tag{4.13}$$

This coincides with eq.(4.3) for $p = 6$ and hence there is no first order phase transition.

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5. Wilson loops and (no) phase transitions

In this section we compute the space-time Wilson loops for the field theories with maximal supersymmetries both at zero and finite temperature. In these calculations we see no trace of phase transitions in the supergravity region. This is in accordance with what was derived in the previous section.

In the four dimensional case, considered in [39], it was found that there are two regions. For $TL \ll 1$ we found a Coulomb behavior while for $TL \gg 1$ the configuration of two parallel strings which end on the brane is energetically favored and, therefore, the force between the quark anti quark vanishes. What we find for the non-conformal cases is exactly the same behavior. This is surprising because the non-conformal theories contain scales associated with the dimensionful g_{YM} in the relevant dimension. This supports, therefore, the conclusion of the previous section that there is no phase transition in field theories with maximal number of supersymmetries in the supergravity region.

Before we calculate the Wilson loop with temperature let us first start with the Wilson line at zero temperature.

5.1. Wilson lines at zero temperature

In [37, 38] a stringy prescription for the computation of the expectation value of the spatial Wilson line of $\mathcal{N} = 4$ was proposed. This expectation value determines the dependence of the energy E of a quark anti-quark pair on the distance between the quarks L. The goal of this section is to extract this dependence for non-conformal theories with 16 supercharges. In fact, one such case, the $\mathcal{N} = 8$ $U(N)$ SYM in 2+1 dimensions was analyzed in [37]. It was found that $E \sim (\frac{g_{YM}^2 N}{L^2})^{1/3}$. It was further shown [37] that this result, which is valid in the supergravity regime, is glued smoothly with the perturbative SYM result, which is valid in the UV. In the very IR there is a third regime where the correct description is in terms of the conformal theory on coinciding M2 branes, but we will not attempt to treat this here. We address now the generalization of this result to other Dp-branes systems.

The worldsheet action that corresponds to the metric G_{MN} of Dp-branes takes the form

$$\begin{aligned} S &= \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{det G_{MN} \partial_\alpha X^M \partial_\alpha X^M} \\ &= \frac{T}{2\pi} \int dx \sqrt{(\partial_x U)^2 + U^{7-p}/R_p^{7-p}}\,, \end{aligned} \tag{5.1}$$

where $R_p = (g_{YM}^2 d_p N)^{1/(7-p)}$.

By repeating the procedure of [37] one finds

$$L = 2R_p(\frac{R_p}{U_0})^{(5-p)/2} \int_1^\infty \frac{dy}{y^{(7-p)/2}\sqrt{y^{7-p}-1}} \sim g_{YM}\sqrt{N} U_0^{(p-5)/2}\,. \tag{5.2}$$

The energy of a system of a quark anti-quark is (for $p \neq 5$)

$$E_{sg} \sim -(\frac{g_{YM}^2 N}{L^2})^{1/(5-p)} , \tag{5.3}$$

the case of $p = 5$ is described separately below. This equation rests on the validity of the 10d supergravity description and hence it is valid as long as the range of U we integrate over is in the supergravity region defined in section 2. This has a double implication: (i) The minimal value of U, U_0, has to be greater than the lower bound stated in (2.4), (ii) since one cannot integrate up to $U = \infty$, to ensure a reasonable approximation one has to demand that $x(U_{ub}) - L/2 \ll L$, where U_{ub} is the upper bound of U. For instance for $p < 3$ the consequence is that (5.3) can be trusted only for

$$(g_{YM}^2 N)^{1/(3-p)} N^{4/(p-3)(7-p)} \ll U \ll (g_{YM}^2 N)^{1/(3-p)} . \tag{5.4}$$

In certain cases discussed below we show that in fact the range can be extended.

Next we show, as in the $2+1$ case [37], that this 10d supergravity result is of the same order as the perturbative SYM result at the transition region.

$$E_{YM} \sim g_{YM}^2 N L^{2-p} . \tag{5.5}$$

The transition occurs at $U \sim (g_{YM}^2 N)^{2/(3-p)}$. Since both energies (eq.(5.5) and eq.(5.3)) and the transition region depend only on $g_{YM}^2 N$, with no separate dependence on N and g_{YM}, it is guaranteed on dimensional grounds, that at the transition, (5.3)

$$E_{sg} \sim E_{YM} \sim g_{YM}^2 N^{\frac{1}{3-p}} . \tag{5.6}$$

To describe the behavior of the various systems in other regions of energy we need to do it for each p separately.

5.2. D1-brane

At first sight, according to (2.4), it seems that (5.3) with $p = 1$ is valid only in the region $g_{YM} N^{1/6} \ll U_0 \ll g_{YM} \sqrt{N}$. In fact the domain of validity of (5.3) can be extended beyond the lower limit $U_0 < g_{YM} N^{1/6}$, where the dilaton becomes large [10]. In the region $g_{YM} \ll U_0 \ll g_{YM} N^{1/6}$ the proper description is via the S-dual system. Namely, the role of the "quark-anti-quark" is played by a D1-string (instead of a fundamental string) and the background is of a collection of N fundamental strings (and not N D1-branes). The action of a D1-string in the background of fundamental strings is the same as the action of F1-string in the background of D1-branes. The reason is that due to the difference between their tension the D1-string action is that of the fundamental string action multiplied with a factor $e^{-\phi}$. On the other hand the S-dual metric contains a factor of e^{ϕ} so that these factors are canceled and one is left with the same expression for the energy of the system. Only at very low energies $U < g_{YM}$ [10] the orbifold conformal theory [47] takes over.

We conclude, therefore, from the supergravity description, that for SYM in $1+1$ dimensions in the region

$$\frac{1}{\sqrt{g_{YM}^2 N}} \ll L \ll \frac{N}{\sqrt{g_{YM}^2 N}} \tag{5.7}$$

the energy between a"quark anti-quark" pair is

$$E \sim (\sqrt{\frac{g_{YM}^2 N}{L^2}})^{1/4}\,. \tag{5.8}$$

5.3. D4-brane

Beyond the region where the ten dimensional supergravity is valid, for $U \gg \frac{N^{2/3}}{g_{YM}^2 N}$, the system is described by M5-branes wrapped on R_{10} [10]. In the case of M5 branes, which was addressed in [37], the role of the "quarks" is played by M2-branes wrapped on R_{10}. The expression for the energy deduced from the "Wilson surface" in the six dimensional (2,0) theory is [37]

$$\frac{E}{L'} \sim -\frac{N}{L^2}\,, \tag{5.9}$$

where L' is the length of the boundary of the M2-brane on the M5-branes. In our case since the M2-brane wrapped the x_{10} direction $L' = 2\pi R_{10} \sim g_{YM}^2$. Thus we obtain the same result as the 10d supergravity expression (5.3) yields for $p = 4$. Thus, in the UV region $L < N g_{YM}^2$ the energy is

$$E \sim -\frac{g_{YM}^2 N}{L^2}\,. \tag{5.10}$$

5.4. D5-brane

In the five-brane case there are only two regions. The SYM region and the 10D supergravity region. Like in the D1-brane case the 10d supergravity region has to be divided into two region. But, again like in the D1-brane case, the two regions, being related by S-duality, yield the same result. The solution of $U(x)$ takes for $p = 5$ the following form $U(x) = U_0/cos(x/R_5)$ which implies that L does not depend on U_0!

$$L = (2\pi)^{-3/2} g_{YM}\sqrt{N} \sim \sqrt{\alpha' N}\,. \tag{5.11}$$

This can also be seen also from (5.2) with $p = 5$.

Let us consider the emerging physical picture. Suppose we start at large distances where we can trust the SYM calculation. The energy of the quark anti-quark is $E \sim -g_{YM}^2 N/L^3$. For $L < g_{YM}\sqrt{N}$ we find that g_{eff} becomes larger then 1 and the supergravity description takes over. Then L is fixed to be $g_{YM}\sqrt{N}$ and one cannot decrease L further. The fact that for $L < g_{YM}\sqrt{N}$ there is no classical geodesic should be interpreted according to [46] as having a zero value of the Wilson loop which implies an infinite potential. However, a non-trivial dependence of U_0 on L, and hence a non-zero Wilson loop, may emerge from a semi-classical quantization of the system with certain collective coordinates. [1] At this point we cannot resist to speculate that perhaps this "classical" minimal distance is related to the existance of a non-locality scale [42].

6. Wilson lines at finite temperature

Next we address the large N behavior of the Wilson line of the non-conformal theories described in the previous section at finite temperature. As in the discussion of the $p = 3$ case this corresponds to a description in terms of a non-extremal supergravity Dp-brane [10] which translates into the following worldsheet action

$$S = \frac{T}{2\pi} \int dx \sqrt{(\partial_x U)^2 + (U^{7-p} - U_T^{7-p})/R^{7-p}}\,. \tag{6.1}$$

The determination of the E as a function of $L, T, g_{YM}\sqrt{d_p N}$ follows the same steps as those leading to (5.3) subjected to the bounds on L mentioned above.

The expression for the energy is

$$E = \frac{U_0}{\pi}\left[\int_1^\infty dy \left(\frac{\sqrt{y^{7-p} - 1 + \epsilon}}{\sqrt{y^{7-p} - 1}}\right) - 1\right] + \frac{(U_T - U_0)}{\pi}\,, \tag{6.2}$$

where $\epsilon = 1 - (U_T/U_0)^{7-p}$. For the length we find

$$L = 2R_p^{(7-p)/2} U_0^{(p-5)/2} \sqrt{\epsilon} \int_1^\infty \frac{dy}{\sqrt{(y^{7-p} - 1)(y^{7-p} - 1 + \epsilon)}}\,. \tag{6.3}$$

At small temperatures where $U_T \ll U_0$ one finds the following result:

$$E(T, L) = c_0 (\frac{g_{YM}^2 N}{L^2})^{1/(5-p)} \left[1 + c(T\frac{L^2}{g_{YM}^2 N})^{(7-p)/(5-p)})\right]. \tag{6.4}$$

The computation of the Wilson line for arbitrary temperature was performed for the case of $p = 3$ in [39]. The results for $1 \le p \le 4$ are similar to that found in [39], for $TL \ll 1$ the quark anti-quark pair is connected by a string and the energy goes as $E \sim -L^{2/(p-5)}$ just like in the zero temperature case. For large $TL \gg 1$ a configuration of two parallel strings going from $U = U_T$ to $U = \infty$ is energetically favored and the force between the quarks vanishes. The length where the string breaks is c/T, where c is a dimensionless constant of order 1 (which does not depend on N, like in the 4d case [39]). We conclude that also in the non-conformal cases we find no phase transitions in the supergravity regime at finite temperature although there is a dimensionful parameter in the theory (in contrast to the conformal p=3 case [39]). Maybe one has to choose a compact space to find a phase transition in field theories

[1] We thank J. Maldacena for pointing this to us.

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with maximal supersymmetries. For p=3 this has been done in [46], where it was found that the four dimensional theory on $S^1 \times R^3$ has no phase transition whereas the theory $S^1 \times S^3$ exhibits a phase transition from confinement at small temperatures to deconfinement at high temperatures (the S^1 denotes the compactified Euclidean time direction with period $\beta = T^{-1}$). But since in the non-conformal case the space-time is not a direct product of AdS_{p+1} times some compact space and we do not know the metrics that would correspond to spaces with boundaries of the form $S^1 \times S^p$ we have to leave the study of compact field theories to the future.

The discussion in this section was restricted to the supergravity and Yang-Mills regimes but one may also try to connect to other regimes [10] e.g. in the p=2 case one needs the M2 brane description in the IR.

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